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# Lane distribution estimation for heterogeneous traffic flows

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## Abstract

A new macroscopic steady state theory is proposed to explain how heterogeneous traffic distributes itself over the lanes of a congested highway. Firstly, a model is derived which predicts the speed of a given mixture of traffic within a single lane. The distribution over lanes is then phrased as an assignment problem, where it is assumed that individual drivers choose lanes so as to try to maximise their own speeds. Theory is derived which establishes circumstances in which the assignment matrix and consequent lane speeds can be solved for. Two examples are presented which demonstrate how the theory can be used to inform traffic management that employs either dynamic speed limits or mandatory lane policies. Future research and applications are then scoped.

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## 1. Introduction

The purpose of this paper is to lay the first blocks in the foundation of a new theory of how heterogeneous traffic flows distribute themselves across multiple lanes of a congested highway. There are a number of practical motivating factors for this work. Firstly, lateral effects in highway traffic are not well studied, yet casual observation during any long highway journey reveals that lane changes are more common than significant longitudinal acceleration events, and moreover, there are patterns in the manner in which different drivers choose to hold different lanes, depending on the type of the vehicle in question, the drivers' personal preferences, and the wider driving norms of the country or region in question. For example, in many European countries, traffic laws instruct drivers to return to the outer lane (left-hand lane in UK, right-hand lane in other countries) after overtaking slower vehicles [UK Department for Transport \(b\)](#). However, this rule is often ignored and in moderately busy conditions, one usually observes that traffic sorts itself into streams of greatly differing speeds, one stream per lane, with very few lane changes between the streams. The recent paper of [Farhi et al. \(2013\)](#), which proposes a simple model for such lane choice based on

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individual drivers' utility, appears to be the only contribution in the literature offering a theory that might help us understand this effect, although [Duret \(2014\)](#) is an other recent related work on lane distribution.

Secondly, Active Traffic Management (ATM) allows the possibility to control traffic on a per-lane basis. A typical set-up (for example, as used in England's Smart Motorways system [UK Department for Transport \(a\)](#)) is to set temporary speed limits by VMS positioned above each individual lane. The usual practice is that all lanes have the same reduced speed limit, but such a system allows in principal that a different speed limit be set for each lane. One might wonder what the effect of such a management scheme would be. In particular, much of the benefit of ATM is thought to derive from an enhancement in stability that results from reducing the variance in the speed of the traffic, both within and between lanes. However, there may be a sub-set of skilled drivers who are able to drive stably at short headways and high speed. Thus if a single lane were to be allowed a higher speed limit, targeted to such drivers, would this increase the capacity of the highway? Other lane management systems work by vehicle class and (for example) include the restriction of trucks to the outermost two lanes of the highway, or alternatively on occasion there may be a lane reserved exclusively for trucks and buses, or indeed one may even set different dynamic speed limits for different vehicle classes.

Thirdly, the introduction of automated vehicles, be they fully autonomous, coordinated by vehicle-to-infrastructure communicated systems, or merely equipped with advanced driver assist systems, is likely to impact significantly on achievable highway capacity in the next 20 years. For example, dedicated lanes for automated vehicles might achieve short headways at high speed (but if this is achieved by taking lanes away from non-automated vehicles, what will the effect on the overall flow be?). Or alternatively, automated vehicles might be instructed by centralised control to penetrate and inter-mix with non-automated vehicles so as to regulate their speed and flow. In which case, what is the optimal way to distribute the automated vehicles across lanes?

Finally, insight into lane choice and (optimal) lane distribution will improve continuum traffic flow models. Some models already take into account multiple lanes and their usage by different types of vehicles ([Hoogendoorn and Bovy, 2001](#); [Daganzo, 2002](#)), other models assign fractions of the road to certain types of vehicles ([Ngoduy and Liu, 2007](#); [Logghe and Immers, 2008](#)). However, to the authors knowledge, the lane distribution is not discussed explicitly in neither of these multi-class models. We did not consider models for behaviour near ramps or other road inhomogeneities as in this first study we are focussing on the (simpler) case of homogeneous roads. Future adaptations of multi-class continuum models including the lane distribution, will be better able to predict, for example, lane specific speeds and flows. Consequently, they will result in better predictions of congestion and class specific travel time.

In this paper, we consider the development of the simplest possible theoretical framework to address these questions *in general*, leaving the practical details for future work. Our basic set-up is that of a homogeneous unidirectional multi-lane highway, which is populated by vehicles belonging to a small number of discrete classes. The idea is that a single *class* contains vehicles which are identical in both their physical characteristics and in terms of their drivers' behaviours.

The simplified situation that we shall examine is one which is static — that is the macroscopic variables of flows, speeds and densities are independent of both time and distance down the highway. Moreover, we shall assume that there are no lane changes, either because traffic is so congested that they cannot occur, or because the traffic management policy specifically prohibits them. (In fact, the theory presented here could be generalised to allow for lane changes where there is no net flow of any one class from one lane to another. In this case, one would introduce a term that penalises capacity in accordance with lane-changing rates.) In this set-up, it is clear that the average speed of every vehicle in the same lane is the same — and thus if we assume that individual vehicles' speeds are time-independent, then each vehicle in the same lane has the same time-independent speed. (NB in fact oscillatory microscopic dynamics is also consistent with the static macroscopic description, but is beyond the simplified scope of this paper.) However, the vehicles' headways will differ according to their class, and indeed the key behavioural property for each class will be an equilibrium speed-spacing function, from which we derive the speed-density properties of a single lane of mixed traffic (Section 2).

The question is then one of traffic assignment: how does a given mixture of traffic distribute itself across the available lanes? Assuming free (non-automated) drivers, the most reasonable model is a kind of user equilibrium, which is explained in more detail in Section 3. Next, Section 4 continues with the main contribution of this paper: a methodology to solve the user equilibrium problem and calculate the distribution of traffic and the speed of each lane,

for a given composition of traffic provided in the form of a density for each class. In Section 5, we then give two short examples which illustrate how our findings might be used to inform traffic management tactics. Finally, Section 6 presents conclusions and discusses possibilities for further research.

## 2. Mixture model for one lane

The key input to the theory that follows is a macroscopic speed-density relation for a single lane multiclass flow which is in equilibrium according to the introductory discussion. There are a number of possible models suggested in the literature (Del Castillo, 2012; Van Wageningen-Kessels et al., 2014a,b), but we follow a ground-up approach which supposes that there is a microscopic speed-spacing function  $V_i(s)$  for each class  $i$ , where  $s$  is the gross spacing measured from the rear of the vehicle in question forwards to the rear of the vehicle that leads it. The assumed *microscopic* equilibrium implies that all vehicles in the same lane have the same common speed  $v$ , so that if there are  $M$  classes  $i = 1, 2, \dots, M$ , then

$$v = V_1(s_1) = V_2(s_2) = \dots = V_M(s_M) \quad (1)$$

where  $s_i$  is the common gross spacing for each vehicle in class  $i$ .

If we suppose that in this lane, each class  $i$  has a macroscopic density  $\rho_i$  (measured in vehicles per unit length of road), then clearly (because all road space must be accounted for by gross spacings of one vehicle or another) we have

$$\sum_i \rho_i s_i = 1. \quad (2)$$

Hence for a single lane, if we suppose class densities  $\rho_i$  are provided as inputs, then (1) and (2) together constitute a system of  $M + 1$  simultaneous equations to solve for the  $M$  class-specific spacings  $s_i$  and the common lane speed  $v$ .

This framework is simplest when the class-specific speed-spacing functions  $V_i$  are invertible, which may be achieved by supposing that each  $V_i$  is continuously differentiable with  $V'_i(s) > 0$  for  $s > s_i^{\text{jam}}$ . In this case, (1) implies each  $s_i$  can be written explicitly in terms of  $v$ , so that (2) is a single scalar equation for the scalar unknown  $v$ , whose solution can be analysed by classical methods (and which if it exists, is unique, since the  $V_i^{-1}$  will also be strictly increasing). However, a natural way to implement a reduced speed limit (a common traffic management measure) is to suppose that a cut-off is applied to the speed-spacing functions so that  $V_i(s) = V_i^{\text{max}}$  for  $s \geq s_i^*$ , rendering them non-invertible. The theory we present in Section 4 can deal with this case also, albeit its full strength is limited to  $M = 2$  classes.

## 3. Multilane framework

We now generalise the single-lane framework to consider a flow with  $M$  classes distributed over  $j = 1, 2, \dots, N$  lanes. As before, the inputs shall be the densities  $\rho_i$  of each class, and the challenge is to model how this density distributes itself over lanes  $j$ . In essence, this is a kind of assignment problem, governed by an assignment matrix with entries  $0 \leq \alpha_{i,j} \leq 1$  so that

$$\rho_{i,j} = \alpha_{i,j} \rho_i, \quad (3)$$

and hence with row sums satisfying

$$\sum_j \alpha_{i,j} = 1, \quad (4)$$

for each class  $i$ . Thus if class densities  $\rho_i$  and an assignment matrix  $\{\alpha_{i,j}\}$  are provided, we have all of the class-specific and lane-specific densities  $\rho_{i,j}$  and the analysis of section 2 may be repeated on a lane-by-lane basis to find the speed  $v_j$  of each of the lanes.

However, the problem that we are most interested in is determining the assignment matrix itself, supposing that the class densities  $\rho_i$  are provided as inputs. The point is that the models of driver behaviour that we will use (or alternatively, the expression of traffic management protocols, system optimal objectives etc.) involve the lane speeds, which are in turn influenced by the assignment matrix.

For example: the simplest possible behavioural model, that we will analyse in detail, is one of user equilibrium, modelling drivers who are free to choose their lane so as to maximise their own speed. It follows that all lanes which are used by a given class  $i$  must end up travelling at the same speed (or there would over time be a net transfer from the slow lane to the fast lane, which would tend to equalise the speeds). Lanes which are unused by class  $i$  must either be slower (hence undesirable) or indeed travelling faster than the speed limit  $V_i^{\max}$  for class  $i$ .

Finally, note that assignment solutions will often be non-unique up to a permutation of the lane numbers. However, in real-world highway traffic, there will be often be an ordering of the lanes in terms of speed and furthermore heavier classes of vehicle will tend to predominate in lower numbered lanes. Hence solutions of our analytical model which are identical up to lane number permutation can be removed by tie-breaking rules that reflect real-world behaviour: firstly by tie-breaking on speed by insisting  $v_1 \leq v_2 \leq \dots \leq v_N$ ; then secondly, if a set of lanes has equal speed, by breaking ties on the concentration of vehicles of class one; then by breaking further ties on the concentration of vehicles of class two — and so on.

#### 4. Underpinning theory and the solution technique for the user equilibrium problem

We now continue with our main contribution, namely a theory that shows how the assignment matrix can be determined. One may view this work as an extension of the single lane results of Section 2 to deal with  $N > 1$  lanes. In what follows, we do not require invertibility of the class-specific speed-spacing functions  $V_i$ . However, a drawback of this approach is that the theory only applies in full to  $M = 2$  classes, with consideration of  $M > 2$  remaining for future work.

**Lemma 1.** *Given the number of lanes  $N$  and the class-specific density  $\rho_i$ , then the spacing  $s_i$  of class  $i$  increases linearly with the fraction of the total number of lanes  $A_i/N$  occupied by class  $i$ .*

*Proof.* Assume a road segment with  $N$  lanes and a given class density of  $\rho_i$ . If there were no vehicles of other classes on the road, then the spacing of class  $i$  would be  $s_i = N/\rho_i$ . Let us now define the number of lanes taken by class  $i$  (the lane distribution variable) as  $A_i$ . Now  $A_i/N$  is the fraction of the total number of lanes occupied by class  $i$ , and its spacing is

$$s_i = \frac{A_i}{N} \frac{N}{\rho_i} = \frac{A_i}{\rho_i}. \quad (5)$$

□

**Lemma 2.** *Given  $N$  lanes, suppose for class  $i$  the following is known: the assignment  $\alpha_{i,j}$ ,  $\forall j \in \{1, \dots, N\}$ , the class-specific density  $\rho_i$ , and the class-specific speed-spacing function  $V_i(s_i)$ . Then the class-specific speed  $v_i$  can be calculated.*

*Proof.* Using Lemma 1, the class specific spacing  $s_i$  can be calculated. Subsequently substituting this into the fundamental diagram  $V_i(s_i)$  gives the class specific speed  $v_i = V_i(s_i)$ . □

So far the theory has been general in the number of classes and lanes. From this point on we restrict to  $M = 2$  classes, although the number of lanes remains general.

**Theorem 1.** *Assume there are  $M = 2$  classes and class 1 has preference for the lanes with the lowest number  $j$ , while class 2 has preference for lanes with the highest number  $j$ . Given  $N$  lanes, the class-specific densities  $\rho_1$  and  $\rho_2$ , and the class specific speed-spacing functions  $V_1(s_1)$  and  $V_2(s_2)$ , then the user equilibrium speeds  $v_1$  and  $v_2$  can be calculated as can the assignment variables  $\alpha_{i,j}$ , according to:*

$$\alpha_{1,j} = \begin{cases} \frac{1}{A_1^{\text{ue}}} & \text{if } j \leq A_1^{\text{full}} \\ \frac{A_1^{\text{ue}} - A_1^{\text{full}}}{A_1^{\text{ue}}} & \text{if } j = A_1^{\text{full}} + 1 \\ 0 & \text{if } j > A_1^{\text{full}} + 1 \end{cases} \quad \alpha_{2,j} = \begin{cases} 0 & \text{if } j \leq A_1^{\text{full}} \\ \frac{A_1^{\text{full}} + 1 - A_1^{\text{ue}}}{N - A_1^{\text{ue}}} & \text{if } j = A_1^{\text{full}} + 1 \\ \frac{1}{(N - A_1^{\text{ue}})} & \text{if } j > A_1^{\text{full}} + 1 \end{cases} \quad (6)$$

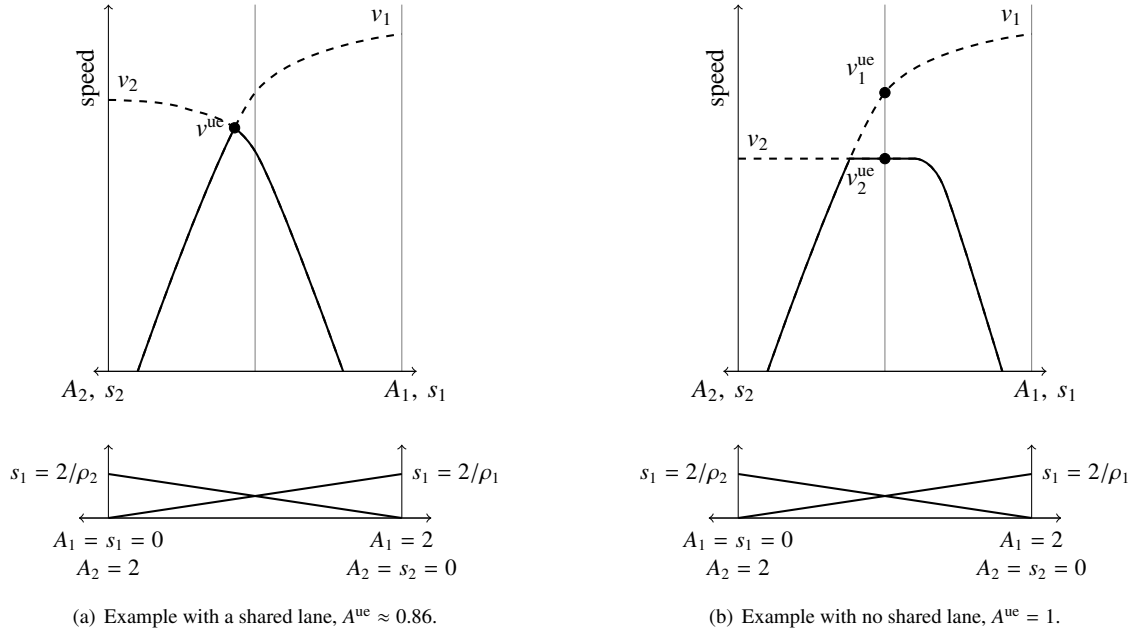


Fig. 1. Illustration of proof of Theorem 1. The lower part in each subfigure shows how the class-specific spacings  $s_1$  and  $s_2$  depend on the lane distribution variables  $A_1$  and  $A_2$ . The upper part shows the speeds corresponding to the lane distributions and spacings (broken lines). The solid line and open dots indicate all feasible speeds. The solid dot indicates the user equilibrium speed. Note that  $A_1$  (and thus also  $s_1$ ) increases from left to right, while  $A_2$  (and thus also  $s_2$ ) increases from right to left

with  $A_1^{\text{full}} = \lfloor A_1^{\text{ue}} \rfloor$  the number of lanes that are fully occupied by class 1 (i.e. there are no vehicles of class 2 in these lanes) and  $A_1^{\text{ue}}/N$  the fraction of lanes taken by class 1 in user equilibrium:

$$A_1^{\text{ue}} = \arg \max_{A_1} \min \left[ V_1 \left( \frac{A_1}{\rho_1} \right), V_2 \left( \frac{N - A_1}{\rho_2} \right) \right]. \quad (7)$$

*Proof.* The proof is illustrated in Figure 1. We first show how the speed is calculated. From Lemma 1 we know that the fraction of road occupied by class 1 maps linearly to its spacing. Since there are only 2 classes, once the number of lanes occupied by class 1 ( $A_1$ ) is known, the number of lanes occupied by class 2 is also fixed by  $A_2 = N - A_1$ . This also implies that there is at most one shared lane, which is reasonable given the assumption of the lane preference of both classes. Once  $A_2$  is known, the spacing of class 2 is also known. Thus the horizontal axis of the graph in Figure 1 is constructed, or to express this in an equation:

$$s_1 = \frac{A_1}{\rho_1}, \quad s_2 = \frac{A_2}{\rho_2} = \frac{N - A_1}{\rho_2} \quad (8)$$

The next step is to find the corresponding class specific speeds corresponding to the spacing:

$$v_1 = V_1(s_1) = V_1 \left( \frac{A_1}{\rho_1} \right), \quad v_2 = V_2(s_2) = V_2 \left( \frac{N - A_1}{\rho_2} \right) \quad (9)$$

These speeds are indicated in Figure 1 with broken lines. However, we made the assumption that if two classes share at least one lane, then their speed is equal. Therefore, if the classes share a lane (i.e., if  $A_1$  is not integer), then the speed of both classes is:

$$v_* = v_*(A_1) = v_1 = v_2 = \min \left[ V_1 \left( \frac{A_1}{\rho_1} \right), V_2 \left( \frac{N - A_1}{\rho_2} \right) \right] \quad (10)$$

These speeds are indicated in Figure 1 with a solid line. If, however, the classes do not share a lane (i.e. if  $A_1$  is integer), then the speeds of both classes may be different, as in (9). These speeds are indicated in Figure 1 with open dots. Finally, the user equilibrium speeds are the maxima of all feasible speeds:

$$\begin{cases} v_1^{\text{uc}} = v_2^{\text{uc}} = v_* (A_1^{\text{uc}}) = \min \left[ V_1 \left( \frac{A_1^{\text{uc}}}{\rho_1} \right), V_2 \left( \frac{N-A_1^{\text{uc}}}{\rho_2} \right) \right] & \text{if } A_1^{\text{uc}} \text{ is not integer (shared lane)} \\ v_1^{\text{uc}} = V_1 \left( \frac{A_1^{\text{uc}}}{\rho_1} \right) \quad \text{and} \quad v_2^{\text{uc}} = V_2 \left( \frac{N-A_1^{\text{uc}}}{\rho_2} \right) & \text{if } A_1^{\text{uc}} \text{ is integer (no shared lane)} \end{cases}$$

$$\text{with } A_1^{\text{uc}} = \arg \max_{A_1} \min \left[ V_1 \left( \frac{A_1}{\rho_1} \right), V_2 \left( \frac{N-A_1}{\rho_2} \right) \right]. \quad (11)$$

These user equilibrium speeds are indicated in Figure 1 with solid dots.

Now the number of lanes taken by class 1 ( $A_1^{\text{uc}}$ ) has been established, we can also determine the assignment matrix. Here we use the assumption that class 1 prefers lanes with low numbers  $j$ , while class 2 prefers lanes with high numbers  $j$ . As discussed before, a permutation is equally valid. We show that the assignment matrix is given by (6) by noting that  $\sum_j \alpha_{i,j} = 1$  holds for both classes  $i = 1$  and  $i = 2$  and by showing that with this assignment matrix, the spacings of the classes  $s_1$  and  $s_2$  are equal on all lanes  $1, 2, \dots, A_1^{\text{full}} + 1$  used by that class:  $s_1 = s_{1,1} = s_{1,2} = \dots = s_{1,A_1^{\text{full}}+1}$ . The proof that the spacing of class 2 is the same on all lanes used by that class ( $s_2 = s_{1,A_1^{\text{full}}+1} = s_{1,A_1^{\text{full}}+2} = \dots = s_{2,N}$ ) is similar and not given here. All lanes  $j = 1, 2, \dots, A_1^{\text{full}}$  are fully occupied by class 1 and thus  $\alpha_{1,j} \rho_1 s_{1,j} = 1$ . We use this and substitute the assignment  $\alpha_{1,j}$  from (6) to find the spacing:

$$s_{1,j} = \frac{1}{\alpha_{1,j} \rho_1} = \frac{A_1^{\text{uc}}}{\rho_1} \quad (12)$$

Lane  $j = A_1^{\text{full}} + 1$  is partially occupied by class 1 and  $\alpha_{1,A_1^{\text{full}}+1} \rho_1 s_{1,A_1^{\text{full}}+1} = A_1^{\text{uc}} - A_1^{\text{full}}$ . Using this and substituting the assignment  $\alpha_{1,A_1^{\text{full}}+1}$  from (6) yields the spacing:

$$s_{1,A_1^{\text{full}}+1} = \frac{A_1^{\text{uc}} - A_1^{\text{full}}}{\alpha_{1,A_1^{\text{full}}+1} \rho_1} = \frac{A_1^{\text{uc}} - A_1^{\text{full}}}{\frac{A_1^{\text{uc}} - A_1^{\text{full}}}{A_1^{\text{uc}}} \rho_1} = \frac{A_1^{\text{uc}}}{\rho_1} \quad (13)$$

□

We now have established a methodology to determine the assignment matrix and the user equilibrium speed. As noted before, the assignment matrix could be permuted, indicating a different lane preference of the vehicles (e.g. class 1 on higher numbered lanes and class 2 on lower numbered lanes). However, this does not change the user equilibrium speed.

## 5. Traffic management examples

In order to illustrate how our theory might be used in practice, we present two examples that model potential traffic management schemes. To simplify matters, our examples consider only  $M = 2$  classes and  $N = 2$  lanes, so that the theory of Section 4 applies in full. It follows that we may use the type of plot introduced in Figure 1, that displays how speeds depend on the lane distribution variables  $A_i$  and spacings  $s_i$ . See Figure 2. The idea behind both these schemes is to separate the flows so that classes do not share lanes, but rather occupy one lane each, with the consequence that the lanes may have different speeds.

### 5.1. Speed limit

In this scheme, the speed limit for one of the classes is set lower than speed that would otherwise be reached at user equilibrium. See Figure 2(a). The user equilibrium is shifted to a lane distribution where the speed-limited class drives at its new maximum speed. The distribution becomes such that neither lane is shared and the classes are

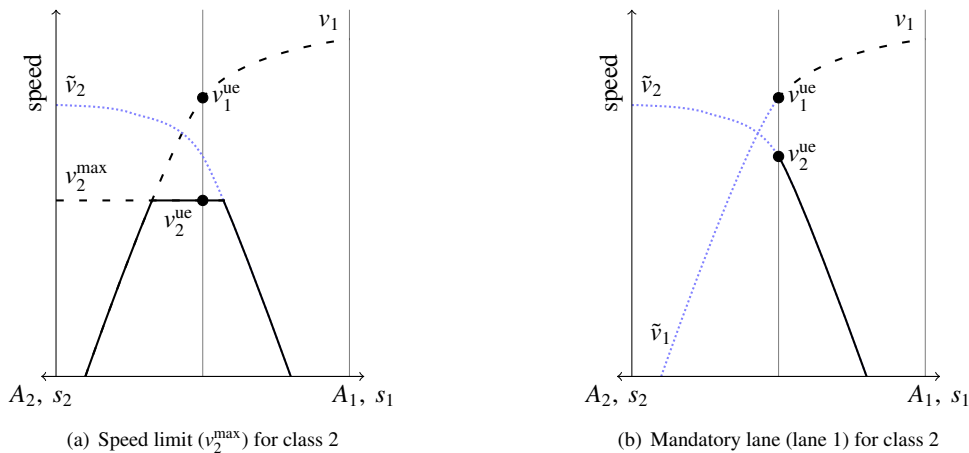


Fig. 2. Illustration of how classes can be segregated by traffic management measures. In both cases class 1 uses only lane 2 and class 2 uses only lane 1. Black dashed lines indicate the speed-spacing relationship with traffic management ( $v_1$  and  $v_2$ ), blue dotted lines indicate speed-spacing relationship without traffic management ( $\tilde{v}_1$  and  $\tilde{v}_2$ ), black solid lines indicate feasible speeds. User equilibrium speeds are indicated with black dots ( $v_1^{ue}$  and  $v_2^{ue}$ ).

separated, each travelling at its own (distinct) speed. As shown in Figure 2(a), this results in a lower speed for class 2 than for class 1. Furthermore, the resulting speed of class 1 is higher than what it would have otherwise been in user equilibrium without traffic management measures (see Figure 1(a)), whereas for class 2 it is slightly lower than without these measures. Of course, one must think about the higher level objectives of the road operator to determine whether this shift is desirable or not.

## 5.2. Mandatory lanes

We suppose that class 2 (heavy vehicles) is required to drive in lane 1, and lane 2 is reserved exclusively for the lighter vehicles of class 1. (One of course could consider more general configurations of mandatory lanes.) See Figure 2(b). The fraction of the road occupied by the class with mandatory lane(s) will decrease, which also results in decreased spacing, which in turn (possibly) decreases speed. However, this leaves more room for the other class, increasing its spacing and thus also allowing it to increase its speed. Just like in the example above with a class-based speed limit, this example results in a lower speed for class 2 than for class 1. However, the speed of class 2 does not decrease as much as it does with the speed limit management method. Moreover, for curves as depicted here, the resulting speed of class 1 is higher than what it would have been in user equilibrium without traffic management measures (see Figure 1(a)); whereas for class 2 it is slightly lower than without the measures.

## 6. Conclusion and further work

In this paper we have built the foundation for a new methodology which may be used to model, explain and predict how different classes of vehicle distribute themselves over the lanes of a highway. The theory is based on a ground-up *mixture model* (Section 2) for multi-class flow on a single lane (itself a worthy result in our view) combined with a traffic assignment idea (Section 3) that each vehicle tries to choose its lane so as to maximise its own speed. At present, we suppose perfect rationality in this choice, and a clear future direction for further research is to use a discrete choice formulation along the lines of Farhi et al. (2013) — the only paper that we can find of direct relevance in this general space.

Our formulation is steady state and does not consider changes in the lane assignment in space and time: however, we anticipate that the theory presented here could be used as an input to a more sophisticated spatiotemporal (PDE) model. However, there is still much that remains to be explored even in the steady state formulation. Solely at the level of modelling, our present paper does not consider the impact of lane changes, which occur microscopically and



cause loss of capacity even when their net lateral effect is zero. This is an obvious and intriguing direction for future research, although it requires a theory of lane-changing rates in terms of macroscopic variables — and that too is an under-researched area with very little empirical data to support it.

In terms of theory, Section 4 is the meat of this paper and develops techniques that support the solution of the user equilibrium lane assignment problem. However, this also is a topic in which more work needs to be done, because in particular, if there are three or more vehicle classes, the problem does reduce to scalar root-finding and consequently we have no general result at present. Furthermore, the theory could be extended to include inhomogeneities such as ramps or lane drops.

A further complication is that of multiple equilibria: a point that we have not elaborated upon in this paper. To explain — consider a two-lane two-class situation somewhat reminiscent of the popular view of the German autobahn, on which a large number of trucks co-exist with a very small number of sports cars. There are two user equilibrium solutions: one is close to the solution where there are no sports cars, in which both classes are mixed together in the same way in both lanes which share the flow equally. This is the ideal situation in terms of capacity, but frustrating if you happen to own a sports car! However, there is also a (highly unstable) equilibrium in which the trucks are crowded into one highly congested lane and the sports cars drive at great speed, far beyond the capability of the trucks, in the other. This sort of discussion leads quite naturally to the question of how one might design traffic management measures so that the user equilibrium solution with measures is close to the system optimal solution without measures.

Here we have just shown just a couple of illustrative examples of how traffic management measures might be implemented in this framework. But much more could be done — for example, consider lane-specific speed limits, more general mandatory lane set-ups, penetration of (externally) controlled driverless vehicles etc.

Finally, the framework shown here is at present set up as an assignment of traffic *density* across the lanes of the highway. This is the simplest thing to do from the point of view of theory, but the set-up should be modified so that instead the assignment operates on the *flows* provided for each class (which are essentially the given traffic demands). This is the very next job in the methodological development, and key to the practical applications, since maximising capacity is usually the main objective of traffic management implementations.

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